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Abstract

Using Exp-function method Öziş and Koroğlu [Öziş T, Koroğlu C., Phys. Lett. A 372 (2008) 3836 - 3840] have found exact ”solutions” of the Fisher equation. In this comment we demonstrate that all these solutions do not satisfy the Fisher equation. The efficiency of application of Exp-function method to search for exact solutions of nonlinear differential equations is questioned by us.

Key words: Nonlinear evolution equation; Fisher equation; Exact solution; Exp-function method; Simplest equation method

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$$u_t - u_{xx} = u(1 - u). \quad (0.1)$$

Eq. (0.1) was studied by Kolmogorov, Petrovskii and Piskunov [3] and by Fisher [4].

Taking the travelling wave $u(x, t) = U(\eta), \eta = k x + w t$ into account authors [1] have presented Eq. (0.1) in the form

$$k^2 U_{\eta\eta} + c w U_{\eta} + U - U^2 = 0, \quad (0.2)$$

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where \(c, w\) and \(k\) are parameters of Eq. (0.2). Solutions of Eq. (0.1) were first found by Ablowitz and Zepetella in [5]. Later solutions of Eq. (0.1) were found many times [6,7].

Using the Exp-function method Özis and Kороğlu [1] found three ”solutions” of Eq. (0.2). These ”solutions” were given in the form

\[
U^{(1)} = \frac{(1 - 2k^2)}{b_0 \exp(\eta) + b_{-1}}, \quad \eta = kx + wt, \tag{0.3}
\]

\[
U^{(2)} = \frac{(1 - 8k^2)}{b_1 \exp(2\eta) + b_{-1}}, \quad \eta = kx + wt, \tag{0.4}
\]

\[
U^{(3)} = \frac{b_0 - 2k^2b_{-1} \exp(-\eta)}{b_0 + b_{-1} \exp(-\eta)}, \quad \eta = kx + wt. \tag{0.5}
\]

However all these ”solutions” do not satisfy equation (0.1). We can note this fact without substitutions solutions (0.3) - (0.5) into Eq. (0.1). The matter is solution of Eq. (0.1) has the pole of the second order but all functions (0.3) - (0.5) with poles of the first order.

To be on the save side we have substituted functions (0.3), (0.4) and (0.5) into Eq. (0.2) and have obtained after multiplying on \((b_0 \exp(\eta) + b_{-1})^3\), \((b_1 \exp(2\eta) + b_{-1})^3\) and \((b_0 + b_{-1} \exp(-\eta))^3\) the following expressions

\[
E^{(1)} = (1 - 2k^2)\left[2k^2b_{-1}^2 + b_0 \left(1 + k^2 - c\ w\right) \left(b_{-1}e\eta + b_0 e^{2\eta}\right)\right], \tag{0.6}
\]

\[
E^{(2)} = (1 - 8k^2)\left[8k^2b_{-1}^2 + b_1 \left(1 + 4k^2 - 2cw\right) \left(b_{-1}e^{2\eta} + b_1 e^{4\eta}\right)\right], \tag{0.7}
\]

\[
E^{(3)} = b_0b_{-1} \left(1 + k^2\right) \left(1 + c\ w - k^2\right) \left(b_0 + b_{-1} e^{-\eta}\right) - 2b_{-1}^3k^2 \left(2k^2 + 1\right) e^{-2\eta}. \tag{0.8}
\]

Taking into account \(w = \frac{1 + k^2}{c}\) in (0.6), \(w = \frac{1 + 4k^2}{2c}\) in (0.7) and \(w = \frac{k^2 - 1}{c}\) in (0.8) we can simplify these expressions but these ones are not equal to zero in the general case and we can see that functions (0.3) - (0.5) do not satisfy Eq. (0.1).

Solitary wave solutions of Eq. (0.1) can be found using the singular manifold method [8–10], the tanh-function method [11, 12], the simplest equation method [13–16] and so on.

Without loss of generality let us apply the simplest equation method [15, 16] to search for exact solutions of the Fisher equation in the form

\[
U_{\eta\eta} + cU_{\eta} + U - U^2 = 0, \tag{0.9}
\]
We assume
\[ U = m_0 + m_1 Y + m_2 Y^2, \quad Y \equiv Y(\eta), \quad (0.10) \]
where \( m_0, m_1, m_2 \) are constants and \( Y(\eta) \) satisfies the Riccati equation in the form
\[ Y_{\eta} = -Y^2 + \beta. \quad (0.11) \]
Taking into consideration the transformation
\[ Y = \frac{\psi_{\eta}}{\psi}, \quad (0.12) \]
we can present formula (0.10) in the form
\[ v = m_0 + m_1 \frac{\psi_{\eta}}{\psi} + m_2 \left( \frac{\psi_{\eta}}{\psi} \right)^2. \quad (0.13) \]
As this takes place Eq. (0.11) can be written in the form of the second-order linear equation
\[ \psi_{\eta\eta} - \beta \psi = 0. \quad (0.14) \]
Substituting (0.10) into (0.9) and equating to zero expressions at different degrees of function \( Y(z) \) we obtain the system of the algebraic solutions with respect to coefficients \( m_0, m_1, m_2 \) and parameter \( \beta \).

We can also apply formulae (0.13) and (0.14) for finding exact solutions of nonlinear differential equations. From comparison formulae (0.10) - (0.14) we can see this is the same approach [16].

Solving the set of the algebraic equations with respect to coefficients \( m_0, m_1 \) and \( m_2 \) we obtain in (0.13)
\[ m_2 = 6, \quad m_1 = -\frac{6}{5} c, \quad m_0 = \frac{1}{2} - 4 \beta - \frac{c^2}{50}, \quad \beta = \frac{c^2}{100} \quad (0.15) \]
\[ c_{1,2} = \pm \frac{5 i \sqrt{6}}{6}, \quad c_{3,4} = \pm \frac{5 \sqrt{6}}{6}. \quad (0.16) \]
Solutions take the form
\[ U = \frac{1}{2} - 4 \beta - \frac{c^2}{50} - \frac{6 c \psi_{\eta}}{5 \psi} + \frac{6 \psi_{\eta}^2}{\psi^2}, \quad (0.17) \]
where
\[ \psi(\eta) = C_1 e^{\sqrt{\beta} \eta} + C_2 e^{-\eta \sqrt{\beta}}, \quad (0.18) \]
where $C_1$ and $C_2$ are arbitrary constants. Taking into account (0.15), (0.16) and (0.18) we have four exact solutions of (0.9) in the form

$$U_{1,2} = \frac{3}{4} \pm \frac{i}{2} \tan \left\{ \frac{\sqrt{6} (\eta - \eta_0)}{12} \right\} + \frac{1}{4} \tan^2 \left\{ \frac{\sqrt{6} (\eta - \eta_0)}{12} \right\}, \quad (0.19)$$

$$U_{3,4} = \frac{1}{4} \left( 1 \mp \tanh \left\{ \frac{\sqrt{6} (\eta - \eta_0)}{12} \right\} \right)^2, \quad (0.20)$$

where $\eta_0$ is arbitrary constant. Substituting (0.19) and (0.20) into Eq. (0.9) at $c = c_{(1,2)}$ and at $c = c_{(3,4)}$ we can convince that (0.19) and (0.20) are solutions of Eq. (0.9).

References