Comment on: "Application of the $(\frac{G'}{G})$ method for the complex KdV equation"[Huiqun Zhang, Commun Nonlinear Sci Numer Simulat 15, 2010, 1700 - 1704]

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Abstract

In this letter we analyze the paper by Zhang [Application of the $(\frac{G'}{G})$ method for the complex KdV equation, Commun Nonlinear Sci Simulat 15 (2010) 1700-1704]. Using the improved $G^{'}/G$ - expansion method Zhang found "new types of exact travelling wave solutions of the complex KdV equation". We checked Zhang's "solutions" and proved that apart from trivial cases none of them satisfy the KdV equation. Moreover, the general solution of the KdV equation in the form of the travelling wave was obtained more than one century ago. We give this solution in this letter for the reference source.

In recent work [1] Zhang looked for exact solution of the complex KdV equation in the form

$$U_t + \mu_1 U U_x + \mu_3 U_{xxx} = 0. \tag{1}$$

Author [1] has used the travelling wave ansatz $U(x,t) = U(z)$, $z = ik(x - ct)$ in Eq. (1) and as a result he obtained the nonlinear ordinary differential equation in the form

$$cU' - \mu_1 U U' + k^2 \mu_3 U''' = 0. \tag{2}$$

The general solution of Eq.(2) was obtained more than century ago [2, 3] but Zhang [1] decided to present "new types of exact travelling wave solutions" of Eq.(2) using the improved $G^{'}/G$ – expansion method. It is known that application of the G'/G – expansion method for finding exact solutions of nonlinear differential equation can not give anything new in comparison with other ansatz methods [4–8]. However, in remark 1 author [1] has written "as we know, these types of exact solutions (24) (solution u_1) and (25) (solution u_2) for the complex KdV equation are new and cannot be found by other existing methods". We agree with author [1] that these "solutions" cannot be found by other methods because they are not solutions.

We substitute Zhang's "solutions" u_1, u_2 and u_3 into Eq.(2) and as a result we have obtained the cumbersome expressions which are not equal to zero. In the case $c_1 = c_2 = 1$ after substituting u_1 we have

$$R_1 = \frac{72\,\mu\,{\mu_3}^2 k^4 \sqrt{\lambda^2 - 4\,\mu}\,(-2\,\mu + \lambda^2)\sinh\left(\sqrt{\lambda^2 - 4\,\mu}\,z\right)}{\mu_1 \left(1 + \cosh\left(\sqrt{\lambda^2 - 4\,\mu}\,z\right)\right)^3}. \tag{3}$$

Expression (3) does not equal zero. So Zhang's "solution" u_1 is not solution of Eq.(2).

Substituting u_2 into Eq.(2) and assuming $c_1 = 0, c_2 = 1, \lambda = 2, \mu = 2$ we obtain the expression

$$R_2 = \frac{576\,(\cos(2\,z) + 3\,\sin(2\,z) + 4)\,k^4 {\mu_3}^2}{\left(1 + (\cos(2\,z))^2 - 2\,\cos(2\,z)\right)\mu_1}. \tag{4}$$

This expression is not equal to zero and the Zhang's "solution" u_2 is not solution as well.

The solution of Eq. (2) has the pole of second order. However the Zhang's "solution" u_3 can be written in the form $u_3 = A\,z^2 + B$, where A and B are some constants. It is obviously that u_3 can not be solution of Eq.(2) because the term $k\,\mu_3\,u^{'''}$ gives zero but the terms $c\,u^{'}$ and $\mu_1\,u\,u^{'}$ are not. There is only the case $c_1 = 0$ when the Zhang's "solution" u_3 is the trivial solution and this solution takes the form

$$u_3 = -\frac{3k^2\mu_3\lambda^2 c_2^2}{\mu_1} + \frac{c + 8k^2\mu_3\mu + k^2\mu_3\lambda^2}{\mu_1}. \tag{5}$$

Let us demonstrate that the general solution of Eq.(2) can be found and we are not going to use the G'/G – expansion method, the Exp - function method and the tanh - function method as well. However, this solution can be obtained by means of the simplest equation method [9, 10].

Note that Eq. (2) can be integrated with respect to z. As a result we have

$$c_1 + cU - \frac{\mu_1}{2}U^2 + k^2\mu_3 U'' = 0. \tag{6}$$

Multiplying Eq. (6) on U' and integrating this equation with respect to z again, we obtain

$$c_2 + c_1 U + \frac{c}{2}U^2 - \frac{\mu_1}{6}U^3 + \frac{k^2\mu_3}{2}U'^2 = 0, \tag{7}$$

where c_1 and c_2 are arbitrary constants. Then we can rewrite Eq.(7) in the form

$$U'^2 = \frac{\mu_1}{3k^2\mu_3}U^3 - \frac{c}{k^2\mu_3}U^2 - \frac{2c_1}{k^2\mu_3}U - \frac{2c_2}{k^2\mu_3}. \tag{8}$$

Taking into account the transformation

$$U = \frac{12k^2\mu_3}{\mu_1}\wp(z) + \frac{c}{\mu_1}, \tag{9}$$

we have equation for the Weierstrass elliptic function

$$(\wp')^2 = 4\,\wp^3 - g_2\,\wp - g_3, \tag{10}$$

where

$$g_2 = \frac{2\mu_1 c_1 + c^2}{12 k^4 \mu_3^2}, \quad g_3 = \frac{3\mu_1(c_1 c + \mu_1 c_2) + c^3}{216 k^6 \mu_3^3}. \tag{11}$$

So, the general solution of Eq. (2) expressed by means of formula (9) via the Weierstrass elliptic function [3, 4].

As for the rational, periodic and solitary wave solutions of Eq. (1) we can obtain all these solutions as the special cases of solution $\wp(z)$. For example, if we take $c_1 = c_2 = 0$ and $c\mu_3 < 0$, we have the solitary wave solution of Eq. (2) in the form

$$U(z) = \frac{3c}{\mu_1}\left(1 - \tanh^2\left(\frac{1}{2k}\sqrt{-\frac{c}{\mu_3}}(z - z_0)\right)\right), \tag{12}$$

where z_0 is an arbitrary constant.

Assuming $c = c_1 = c_2 = 0$ (in this case $g_2 = g_3 = 0$) we have the rational solution of Eq.(2) in the form

$$U(z) = \frac{12 k^2 \mu_3}{\mu_1 (z - z_0)^2}. \tag{13}$$

In the conclusion we can note that author [1] made some common errors that were discussed in a number of the recent works [4–8, 11–15].

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